Flow Regimes in Snow Avalanches: New Insights and Their Possible Consequences*

D. Issler†

1 A brief (and biased) review of experimental knowledge on avalanches

Snow avalanches occur in a bewildering variety of sizes and forms. Flow depths range from 10^{-1} m to 10^2 m, runout distances from 10^1 m to 10^4 m, flow velocities from $10^0\,\mathrm{m\,s^{-1}}$ to $10^2.\,\mathrm{m\,s^{-1}}$, and masses from 10^0 t to 10^6 t. The flowing material may be dry, humid, wet (or even mixed with water in the case of slush avalanches). Humid or wet avalanches usually form shallow flows (relative to their length and width) of quite high density, with moderate to low agitation of the constitutive particles. Dry-snow avalanches instead often develop a deep layer of suspended particles that may completely separate from the dense avalanche core and follow a different path. For general reviews of avalanche dynamics, see (Hopfinger, 1983; Hutter, 1996; Issler, 2003).

Besides the density, the particle size distribution and the water content are important factors determining the rheological behavior of snow in motion. In typical dry-snow avalanche deposits, snow balls of various sizes (from less than 1 cm to nearly 1 m) are embedded in a matrix of fine-grained snow (Issler et al., 1996). The deposits are often quite cohesive, but this may be a property developed only in the very last phase of the motion. The flowing dry-snow avalanche shares many properties with polydisperse granular materials. Some wet-snow avalanches clearly resemble granular flows as well, but there are other cases where visco-plastic behavior is predominant.

Typical densities in dry dense-flow avalanches are believed to range from 100 to $300\,\mathrm{kg\,m^{-3}}$. Particles are then more or less permanently in contact with others so that stresses are transmitted either by chains of particles in contact (the so-called effective stresses) or through collisions of particles. The mean free path is below one particle diameter in this case. In contrast, the density of the suspension layer (the powder-snow avalanche) is below $10\,\mathrm{kg\,m^{-3}}$, i. e., less than 1 % of the volume is occupied by snow grains. The mean free path is many particle diameters, particle collisions are very rare and stresses are transmitted mainly by aerodynamic forces. Impact measurements using sufficiently small load cells show very high continuous pressures (up to about 1 MPa) in the dense core of an avalanche; pressure fluctuations in the tens of millisecond range are significant,

*Invited talk presented at the conference "L'ingegneria e la neve" of the Associazione Georisorse e Ambiente, Politecnico di Torino, Torino (Italy), 21 February 2006.

†NaDesCoR, Seeweg 27, CH–8852 Altendorf, Switzerland. E-mail: *nadescor@bluewin.ch.*

but clearly less than the mean pressure (Schaerer and Salway, 1980; Nishimura et al., 1987; Schaer and Issler, 2001). In the suspension layer, pressures are much lower (a few kPa) and show fluctuations in the range close to one second, characteristic of turbulent flow (Nishimura, 1995).

For at lest 25 years, there has been evidence for an intermediate flow regime in dry-snow avalanches (Schaerer and Salway, 1980; Nishimura et al., 1987; Schaer and Issler, 2001). Its hallmark are very high pressure peaks (again up to 1 MPa) lasting for about ten milliseconds, between which the pressure drops to almost zero. These peaks correspond to the impacts of single snow particles of various sizes. From the impact frequency and impulse, one infers densities between 10 and 100 $\mathrm{kg\,m^{-3}}$in this layer (Schaer and Issler, 2001). In measurements with load cells and radar it may be found both ahead of the dense core and above it. Observations of medium-size to large dry-snow avalanches very often reveal an extended distal runout zone where the deposits are shallow and covered with scattered snow blocks whose size may approach 1 m (Issler et al., 1996). On the basis of the experimental observations, one infers a free mean path of a few particle diameters. Hence both particle collisions and inertial transport of momentum must be important in stress generation.

Given the visibly granular nature of many snow avalanches, recent novel laboratory studies of the flow of dry granular materials (Pouliquen, 1999; Pouliquen and Forterre, 2002; Louge and Keast, 2001; GDR MiDi, 2004; Rajchenbach, 2000, 2004) may be relevant to avalanche modeling. Steady granular flows are possible only in a limited range of slope angles that moreover depend on the flow depth. The flow velocity grows with the 3/2 power of the flow depth. These findings imply that the friction law for granular flows is much more complicated than the simple Coulomb law $F_f = \mu F_n$ that is often assumed (F_f and F_n are the friction and normal forces, respectively, μ is the friction coefficient). Detailed measurements in very densely packed flows revealed *linear* velocity profiles—a finding that cannot be explained in traditional rheological terms (Rajchenbach, 2004).

2 Avalanche models

Avalanche dynamics is a fascinating topic for scientific research in its own right, but the main driving force has been the practical need of protecting settlements and traffic routes. Here, the main concern is not so much the time of release of frequent events (which is important for temporary measures such as road closures and evacuations), but the reach of rare and extreme avalanches (frequency $\ll 10^{-1}\,\mathrm{a}^{-1}$) and their effect on structures. *Hazard mapping* is the primary area where avalanche models are applied, but they may also help greatly in the *dimensioning of protective structures* such as deflection or catching dams—if they are advanced enough to handle such complicated geometries. Accordingly, dynamical models of widely disparate complexity have been developed. We shall not consider the statistical runout models here because they do not give information on pressure. A condensed and useful overview of avalanche models up to 1997 can be found in (Harbitz, 1998), much of the pioneering Russian work is summarized in (Eglit, 1983, 1998).

The simplest and also earliest models that have been proposed treat the avalanche as a sliding point mass. Various laws for the bottom friction can be implemented, and the solution for the center-of-mass motion can be expressed in terms of a quadrature. A variant has been used for powder-snow avalanches: the mass "point" is given a length, width and height, which vary according to dynamical or geometric criteria and may influence the resistive forces (Fukushima and Parker, 1990; Beghin and Olagne, 1991).

At the other extreme of the scale are the models that describe the avalanche as an assembly of particles and solve their equations of motion, including collisions. Such models are not presently used for modeling an entire event from release to stop, but to study the influence of the particle properties on the rheology of the granular material. An interesting attempt to introduce some degree of granularity (and stochasticity) is the PLK model (Perla et al., 1984), which is routinely used in North America.

For the description of powder-snow avalanches, the most complete models solve the three-dimensional Navier-Stokes equations with extra equations for the evolution of turbulence and the motion of suspended snow particles (Hermann et al., 1994; Naaim and Gurer, 1998; Zwinger et al., 2003). Depending on the model, various assumptions are made for the interaction with the snow cover (entrainment and deposition), and between the dense core and the suspension layer. These codes can only be used by specially trained professionals.

Descendants of the shallow-water equations of hydraulics are presently the favored compromise between accuracy and speed, particularly for dense-snow avalanches. The balance equations for the mass and the momentum are integrated over the flow depth and have the following structure:

$$\frac{\partial h}{\partial t} + \frac{\partial (hU)}{\partial x} = w_e, \tag{1}$$

$$\frac{\partial (hU)}{\partial t} + \frac{\partial (fhU^2)}{\partial x} = \frac{1}{\rho}\left[\frac{\partial (h\overline{\sigma}_x)}{\partial x} + \tau_t - \tau_b\right] + g\sin\theta(x). \tag{2}$$

$h(x,t)$ and $U(x,t)$ are the flow depth and the depth-averaged velocity, respectively, $\overline{\sigma}_x$ is the depth-averaged longitudinal stress, τ_t and τ_b are the shear stresses at the top and bottom surface. ρ, g and θ are the density, gravitational acceleration and slope angle, respectively. The Boussinesq coefficient f is close to 1 and depends on the velocity profile. Finally, w_e is the speed at which the snow cover is entrained; most models neglect it, some assume it to depend on the snow properties and the bottom shear stress exerted by the avalanche. These equations are written for the one-dimensional case (coordinate x along the centerline of the path), but can easily be extended to 2D.

The two most popular friction laws are Coulomb friction, $\tau_b^{(C)} = \mu h\rho g\cos\theta$, and Coulomb friction combined with a "turbulent" drag term, $\tau_b^{(V)} = \tau_b^{(C)} + k\rho U^2$, as introduced by Voellmy (1955). The Coulomb model is scale-invariant and predicts ever-accelerating flows on slopes with inclination $\tan\theta > \mu$. The Voellmy model predicts a terminal velocity that scales as the square root of the flow height. The NIS model (Norem et al., 1987, 1989) postulates dispersive stresses proportional to the square of the shear rate and predicts a terminal velocity scaling as $h^{3/2}$, in agreement with at least some of the granular flow experiments. A more radical proposal has recently been made by Naaim et al. (2004) on the basis of chute experiments with natural snow: An empirical friction law is derived directly from the experimental results (after appropriate non-dimensionalization in terms of the Froude number $\mathrm{Fr} = U/\sqrt{gh}$). The thin bottom layer of very high shear is represented in terms of a slip velocity, above it a linear velocity profile is assumed. The same data is used to model snow entrainment and deposition in a way that takes into account the acceleration of eroded material.

3 Modeling flow-regime transitions

The transition from the dense-flow regime to the fluidized one has not been directly observed in snow avalanches. On the basis of simple estimates, we may exclude a purely aerodynamic mechanism that would be similar to hydroplaning in subaqueous debris flows (Elverhøi et al., 2000; Mohrig et al., 1999). The most viable candidate mechanism leading to fluidization is therefore the dispersive pressure created by particle collisions, even though these collisions are quite inelastic for snow blocks. Extending the approach of the NIS model, we assume the normal stresses to be the combined effect of continuous particle contacts (the effective stress p_e) and dispersive pressure proportional to the square of the shear rate $\dot{\gamma}(z) = \partial u(z)/\partial z$,

$$\sigma_z = -p_e - \rho m_2(\rho)\dot{\gamma}^2. \tag{3}$$

Similarly, the shear stress is made up of Coulomb dry friction and a dispersive component,

$$\tau = \mu p_e + \rho m(\rho)\dot{\gamma}^2. \tag{4}$$

The coefficients m_2 and m are functions of the density, as will be discussed below.

In the simplest case of steady shear flow on a slope inclined at a constant angle θ, the balance of forces demands $\sigma_z = -(h-z)\rho g \cos\theta$ and $\tau = (h-z)\rho g \sin\theta$. Inserting this into Eqs. 3 and 4 and solving for $\dot{\gamma}$ and p_e, we obtain

$$\dot{\gamma}(z) = \left[(h-z)\frac{g(\sin\theta - \mu\cos\theta)}{m - \mu m_2}\right]^{1/2}, \tag{5}$$

$$p_e(z) = (h-z)\rho g \cos\theta \frac{m - m_2\tan\theta}{m - \mu m_2}. \tag{6}$$

For slope angles $\tan\theta \geq m/m_2$, the effective pressure vanishes—the hallmark of fluidization as the whole overburden weight is supported by the pressure due to particle collisions (and inertial momentum transport).

What happens when the fluidization threshold is crossed, e. g. after a steepening of the path? The dispersive pressure drives the particles apart, the density decreases, and so does the frequency of collisions: both m and m_2 decrease when the density decreases. If their ratio remained constant thereby, the dilution would never stop. But if m decreases more slowly than m_2 with diminishing density, the ratio $m(\rho)/m_2(\rho)$, which is the effective friction coefficient, increases as the avalanche dilutes. A new equilibrium density $\rho_*(\theta)$ is reached when

$$\frac{m(\rho_*)}{m_2(\rho_*)} = \tan\theta. \tag{7}$$

Note that the fluidized density will be much lower than the non-fluidized one if the ratio m/m_2 decreases slowly with increasing density.

The rheology embodied by the NIS model is rather general and neither predicts the values of the coefficients μ, m and m_2 nor their density dependence. Particle models of granular materials may lead us one step further, but there are many such models with conflicting predictions. Early theories of granular matter based on kinetic theory did not find a density dependence of the effective friction, but extensive computer simulations based on a 2D hard-sphere model (Campbell and Gong, 1986) indicated a pronounced decrease of μ_{eff} with increasing density. An analytic study of the kinetic theory of dense granular gases in 2D that paid particular attention to the geometry of the collisions in the averaging process (Pasquarell et al., 1988) revealed the likely reason for this density dependence. It is the combined effect of the directional dependence of collisional momentum transfer for a single collision, the directional dependence of the collision probability, and the density dependence of the collision velocity (at fixed shear rate). It therefore seems that the effective friction coefficient indeed has the behavior that leads to a new equilibrium density in the fluidized phase. However, the precise form of the density dependence of m and m_2 cannot be inferred from these 2D studies.

The resulting model thus has the following form:

- The balance equations for mass and momentum have a similar form as in the original NIS model.
- A subroutine checks after each time step whether the fluidization conditions are reached somewhere in the flow. Where this is the case, the current equilibrium density is determined using a prescribed curve $\rho_*(\theta)$.
- Where the density differs from the equilibrium density, it is adjusted according to

$$\rho(t+dt) = \rho(t) + \frac{dt}{T}(\rho_* - \rho(t)), \tag{8}$$

  with dt the length of the timestep and T a suitably chosen relaxation time. The flow height is also modified for mass conservation:

$$h(x,t) \longrightarrow h(x,t) \cdot \frac{\rho(t)}{\rho(t+dt)}. \tag{9}$$

A second-order accurate, conservative non-oscillatory central scheme is currently being implemented to solve these equations.

4 Do we need such sophisticated models?

Advanced models like the one presented in the preceding section contain a considerable number of adjustable parameters, yet they are not complete enough to predict the values of these parameters. Therefore a corresponding amount of experimental data is needed to determine the probable ranges of these parameters. Among avalanche researchers, there are two opposite attitudes towards this problem: One group considers that our knowledge of avalanche dynamics will always be insufficient and therefore advocates the use of the simplest models with three or fewer adjustable parameters that are to be calibrated extensively. The price to pay is a very wide range of these parameters that are moreover nearly devoid of precise physical meaning (Barbolini et al., 2000; Issler et al., 2005). Prime examples are the Voellmy–Salm and Perla–Cheng–McClung models (Voellmy, 1955; Salm et al., 1990; Bartelt et al., 1999; Perla et al., 1980).

The opposite attitude is to try to construct models that correctly capture the main physical processes in avalanche flow and contain parameters with a clear physical meaning. Advocates of this approach argue that the parameters can in principle be measured in experiments and their probable range of values can be guessed in advance. This is perhaps most clearly realized in a powder-snow avalanche model proposed by the author (Issler, 1998). The new extended NIS model also belongs into this category because its parameters can in principle be measured in laboratory experiments. Moreover, one may expect that molecular dynamics simulations of small portions of the flow can be used to determine the correct parameter values from first principles.

Finally, there are a few models that do not readily fall in either category: The Savage–Hutter model (Savage and Hutter, 1989; Pudasaini and Hutter, 2004) has only two parameters that are directly measurable and have precise meaning in the context of dry granular flows, but it neglects a great many processes that are specific to, and important in, snow avalanches. The new model proposed by Naaim et al. (2004) gives complete precedence to (laboratory) experimental data; it does not seek direct phyical intgerpretation of the empirical friction law, yet it does not in principle require calibration by means of field data.

In the rest of this paper we adopt a pragmatic point of view and ask what consequences are to be expected if we use the proposed extended NIS model? This model assigns rather different properties to two components of the avalanche that are not distinguished by other models. In an avalanche flow, the head typically has a much higher velocity than the tail. Accordingly, fluidization will usually occur only in the front part of the avalanche. The model states that the parameter m that determines the shear stress is smaller in the fluidized phase than in the dense phase. Therefore, the shear rate will be higher than it would (hypothetically) be in the dense phase under the same conditions. Since the flow height also is larger, a significantly higher velocity results at lower density. In many situations, the fluidized phase will attain a much longer runout distance than the dense phase, and due to its high velocity, it may follow a different path, e. g. lspilling over the bend of a gully. Examples of such behavior have been observed many times (Issler et al., 1996).

It is too early to predict with confidence how strongly hazard maps created with the new model will differ from the existing ones. After all, in most cases those maps were not elaborated by blindly applying a numerical model following rigid guidelines, but also by taking into account observations of past events and the expert's judgment. Nevertheless, the conventional models were calibrated using rare or extreme events, which often will generate a fluidized phase. It is the runout distance of the fluidized phase that was used for the calibration, resulting in very low friction coefficients. Running the conventional models with these parameters, the correct runout distance is obtained, but much too large deposits are predicted in the far distal area, where, in fact, only relatively thin deposits due to the fluidized phase are observed. Furthermore, pressures in the distal area are calculated with an assumed density typical of the dense core (a value of $300\,\mathrm{kg\,m^{-3}}$ is suggested in the Swiss guidelines), whereas the density of the fluidized phase is smaller by a factor of 3–10. With the new two-flow-regime model, one may therefore expect the following differences compared to the conventional models:

- In avalanches where fluidization does not occur, the effective friction may be larger than in the conventional models, so the runout distances may be somewhat shorter.
- If fluidization does occur, the fluidized phase attains higher velocities than predicted by the conventional models and may take a straighter path than expected for a slower

avalanche. This is important in winding channels or on alluvial fans with an incised channel that is followed by the dense phase, but not by the fluidized one.

- On a valley floor, the fluidized phase stops near the same point or slightly beyond the runout of the conventional models while the dense phase comes to a halt more proximally. Beyond the runout of the dense core, the new model is expected to predict pressures that are lower by a factor ≥ 2 than those obtained hitherto.

- The effects on the zoning depend on the criteria that differ from one country to another. Generally, the zones of moderate danger might be extended somewhat in cases where truly extreme avalanches have not been observed yet and the zoning depends on modeling alone. In contrast, the zones of high danger will be reduced in many cases.

However, the readers are cautioned that these conclusions are still rather speculative. Extensive testing of the model is needed before it can be used routinely in hazard mapping, and more high-quality data from the two large avalanche test sites at Ryggfonn (Norway) and Vallée de la Sionne (Switzerland) is eagerly awaited.

Acknowledgments

Much of the work leading to this paper was funded by the European Commission through the project SATSIE (Avalanche Studies and Model Validation in Europe, contract EVG1–2001–00055), the Norwegian Research Council, and the Norwegian Geotechnical Institute. Funding for the writing came from the Swiss National Science Foundation, grant no. 200021–101911. Many colleagues have contributed to the development of the ideas summarized here; I am particularly indebted to Peter Gauer, Carl Harbitz, Fridtjov Irgens and Arne Moe for discussions on the transition to fluidized flow.

References

Barbolini, M., U. Gruber, C. J. Keylock, M. Naaim, and F. Savi. 2000. Application of statistical and hydraulic-continuum dense-snow avalanche models to five real European sites. *Cold Regions Sci. Technol.*, **31**, 133–149.

Bartelt, P., B. Salm, and U. Gruber. 1999. Calculating dense snow avalanche runout using a Voellmy-fluid model with active/passive longitudinal straining. *J. Glaciol.*, **45**(150), 242–254.

Beghin, P. and X. Olagne. 1991. Experimental and theoretical study of the dynamics of powder snow avalanches. *Cold Regions Sci. Technol.*, **19**, 317–326.

Campbell, C. S. and A. Gong. 1986. The stress-tensor in a two-dimensional granular shear flow. *J. Fluid Mech.*, **164**, 107–125.

Eglit, E. M. 1983. Some mathematical models of snow avalanches. *In* Shahinpoor, M., editor, *Advances in the Mechanics and the Flow of Granular Materials*, volume II. Clausthal-Zellerfeld, Germany, Trans Tech Publications, 1st edition, pages 577–588.

Eglit, M. E. 1998. Mathematical and physical modelling of powder-snow avalanches in Russia. *Annals Glaciol.*, **26**, 281–284.

Elverhøi, A., C. B. Harbitz, P. Dimakis, D. Mohrig, J. Marr, and G. Parker. 2000. On the dynamics of subaqueous debris flows. *Oceanogr.*, **13**(3), 109–117.

Fukushima, Y. and G. Parker. 1990. Numerical simulation of powder-snow avalanches. *J. Glaciol.*, **36**(123), 229–237.

GDR MiDi, Groupement de recherche Milieux divisés. 2004. On dense granular flows. *Eur. Phys. J. E*, **14**(4), 341–365; doi:10.1140/epje/i2003–10153–0.

Harbitz, Carl. 1998. A survey of computational models for snow avalanche motion. NGI Report 581220–1. Postboks 3930 Ullevål Stadion, N–0806 Oslo, Norway, SAME Collaboration / Norges Geotekniske Institutt.

Hermann, F., D. Issler, and S. Keller. 1994. Towards a numerical model of powder snow avalanches. *In* Wagner, S., E. H. Hirschel, J. Périaux, and R. Piva, editors, *Proc. of the Second European Computational Fluid Dynamics Conference, Stuttgart (Germany), September 5-8, 1994.* Chichester, UK, J. Wiley & Sons, Ltd., pages 948–955.

Hopfinger, E. J. 1983. Snow avalanche motion and related phenomena. *Ann. Rev. Fluid Mech.*, **15**, 47–76.

Hutter, K. 1996. Avalanche dynamics. *In* Singh, V. P., editor, *Hydrology of Disasters.* Dordrecht/Boston/London, Kluwer Academic Publishers, pages 317–394.

Issler, D. 1998. Modelling of snow entrainment and deposition in powder-snow avalanches. *Annals Glaciol.*, **26**, 253–258.

Issler, D. 2003. Experimental information on the dynamics of dry-snow avalanches. *In* Hutter, K. and N. Kirchner, editors, *Dynamic Response of Granular and Porous Materials under Large and Catastrophic Deformations*, volume 11 of *Lecture Notes in Applied and Computational Mechanics.* Berlin, Germany, Springer, pages 109–160.

Issler, D., P. Gauer, M. Schaer, and S. Keller. 1996. Staublawinenereignisse im Winter 1995: Seewis (GR), Adelboden (BE) und Col du Pillon (VD). Internal Report 694. CH–7260 Weissfluhjoch/Davos, Switzerland, Eidg. Institut für Schnee- und Lawinenforschung.

Issler, D., C. B. Harbitz, K. Kristensen, K. Lied, A. S. Moe, M. Barbolini, F. V. De Blasio, G. Khazaradze, J. N. McElwaine, A. I. Mears, M. Naaim, and R. Sailer. 2005. A comparison of avalanche models with data from dry-snow avalanches at Ryggfonn, Norway. *In* Senneset, K., editor, *Proc. 11th Intl. Conference and Field Trip on Landslides, Norway, 1–10 September, 2005.* Netherlands, A. A. Balkema, Taylor & Francis Group, page in press.

Louge, M. Y. and S. C. Keast. 2001. On dense granular flows down flat frictional inclines. *Phys. Fluids*, **13**(5), 1213–1233.

Mohrig, D., A. Elverhøi, and G. Parker. 1999. Experiments on the relative mobility of muddy subaqueous and subaerial debris flows, and their capacity to remobilize antecedent deposits. *Marine. Geol.*, **154**, 117–129.

Naaim, M. and I. Gurer. 1998. Two-phase numerical model of powder avalanche—theory and application. *Natural Hazards*, **117**, 129–145.

Naaim, M., F. Naaim-Bouvet, T. Faug, and A. Bouchet. 2004. Dense snow avalanche modeling: flow, erosion, deposition and obstacle effects. *Cold Regions Sci. Technol.*, **39**, 193–204.

Nishimura, K. 1995. Measurements of powder snow avalanches—nature. *Surveys Geophys.*, **16**, –660.

Nishimura, K., N. Maeno, and K. Kawada. 1987. Internal structure of large-scale avalanches revealed by a frequency analysis of impact forces. *Low Temp. Sci, Ser. A*, **46**, 91–98.

Norem, H., F. Irgens, and B. Schieldrop. 1987. A continuum model for calculating snow avalanche velocities. *In* Salm, B. and H. Gubler, editors, *Avalanche Formation, Movement and Effects. Proceedings of the Davos Symposium, September 1986*, IAHS Publication No. 162. Inst. of Hydrology, Wallingford, Oxfordshire OX10 8BB, UK, IAHS Press, pages 363–380.

Norem, H., F. Irgens, and B. Schieldrop. 1989. Simulation of snow-avalanche flow in run-out zones. *Annals Glaciol.*, **13**, 218–225.

Pasquarell, G. C., N. L. Ackermann, H. H. Shen, and M. A. Hopkins. 1988. Collisional stress in granular flows: Bagnold revisited. *J. Eng. Mech.*, **114**(1), 59–64.

Perla, R., T. T. Cheng, and D. M. McClung. 1980. A two-parameter model of snow avalanche motion. *J. Glaciol.*, **26**(94), 197–207.

Perla, R., K. Lied, and K. Kristensen. 1984. Particle simulation of snow avalanche motion. *Cold Regions Sci. Technol.*, **9**, 191–202.

Pouliquen, O. 1999. Scaling laws in granular flows down rough inclined planes. *Phys. Fluids*, **11**(3), 542–548.

Pouliquen, O. and Y. Forterre. 2002. Friction law for dense granular flows: application to the motion of a mass down a rough inclined plane. *J. Fluid Mech.*, **453**, 133–151.

Pudasaini, S. P. and K. Hutter. 2004. Dynamics of avalanches along general mountain slopes. *Annals Glaciol.*, **38**, 357–362.

Rajchenbach, J. 2000. Granular flows. *Adv. Phys.*, **49**(2), 229–256.

Rajchenbach, J. 2004. Some remarks on the rheology of dense granular flows: A Commentary on "On dense granular flows" by GDR MiDi. *Eur. Phys. J. E*, **14**(4), 367–371; doi:10.1140/epje/i2004–10025–1.

Salm, B., A. Burkard, and H. U. Gubler. 1990. Berechnung von Fliesslawinen. Eine Anleitung für Praktiker mit Beispielen. Mitteil. EISLF 47. CH–7260 Davos-Weissfluhjoch, Eidg. Institut für Schnee- und Lawinenforschung (EISLF/SFISAR).

Savage, S. B. and K. Hutter. 1989. The motion of a finite mass of granular material down a rough incline. *J. Fluid Mech.*, **199**, 177–215.

Schaer, M. and D. Issler. 2001. Particle densities, velocities, and size distributions in large avalanches from impact-sensor measurements. *Annals Glaciol.*, **32**, 321–327.

Schaerer, P. A. and A. A. Salway. 1980. Seismic and impact-pressure monitoring of flowing avalanches. *J. Glaciol.*, **26**(94), 179–187.

Voellmy, A. 1955. Über die Zerstörungskraft von Lawinen. *Schweiz. Bauzeitung*, **73**, 159–165, 212–217, 246–249, 280–285.

Zwinger, T., A. Kluwick, and P. Sampl. 2003. Numerical simulation of dry-snow avalanche flow over natural terrain. *In* Hutter, K. and N. Kirchner, editors, *Dynamic Response of Granular and Porous Materials under Large and Catastrophic Deformations*, volume 11 of *Lecture Notes in Applied and Computational Mechanics*. Berlin, Germany, Springer, pages 161–194.